REVISTA DE LA FACULTAD DE INGENIERIA
www.revistaingenieria.uda.cl

32 (2016) 1-9

ANÁLISIS DEL GOLPE DE ARIETE USANDO EL MÉTODO DE LAS CARACTERÍSTICAS

John Twyman Q.
Twyman Ingenieros Consultores.
Pasaje Dos # 362, Rancagua, Chile
Persona con quien sostener la correspondencia. Celular: (+56 9) 89044770, Mail: john@twyman.cl, Dirección: Pasaje Dos # 362, Rancagua, Región del Libertador, Chile.
E-mail: john@twyman.cl

ABSTRACT

The Method of Characteristics (MOC) gives an exact result only when it satisfies the Courant condition (C_n). The increment of $C_n < 1.0$ up to the optimal value $C_n = 1.0$ means modify some of the parameters that define its value, that is pipe length, number of reaches or wave speed. The increment of the time step is outside the possibility of change because a slight variation of its value could alters the discretization adopted for the whole system composed by hundreds or thousands of pipes. Another way is to leave everything unchanged and apply a numerical interpolation method which could degrade solution. It will show that MOC is unable to solve water hammer in an extremely simple but numerically problematic pipe network, leaving exposed some weaknesses inherent in the method, such as the need to modify, before its application, some initial data in order that may function properly.
KEYWORDS: Method of the Characteristics, Water Hammer, Pipe Network Discretization.

RESUMEN

El Método de las Características (MC) es exacto sólo cuando cumple la condición de Courant (C_n). Cuando $C_n \neq 1{,}0$, una forma de acercar C_n al valor 1.0 pasa por modificar la longitud de la tubería, la velocidad de la onda o la cantidad de sub−tramos, según sea el caso. Modificar el paso de tiempo queda fuera de opción porque una leve variación en su valor alteraría la discretización adoptada para una red compuesta de decenas o cientos de tuberías. Otro camino es dejar todo inalterado y aplicar un método de interpolación numérica que, de todas formas, podría degradar la solución. Se mostrará que el MC es incapaz de resolver el golpe de ariete en una red extremadamente simple aunque numéricamente problemática, dejando de manifiesto algunas debilidades inherentes al método, como la necesidad de modificar, antes de su aplicación, algunos datos iniciales en orden a que pueda funcionar correctamente.

PALABRAS CLAVE: Método de las Características, Golpe de Ariete, Discretización de la Red de Tuberías.

1. INTRODUCTION

For many years the Method of the Characteristics (MOC) has been used for solving the transient flow in pipe networks due to its numerical efficiency, computational accuracy, and programming simplicity. However, one difficulty that arises is the selection of appropriate time step (Δt) to use for the analysis. The challenge of selecting a time step is made difficult in pipeline systems because to calculate head and discharge in many boundary conditions it is necessary that the time step be common to all pipes. Besides, MOC requires that ratio of the distance step Δx to the time step (Δt) be equal to the wave speed “a” in each pipe, or that Courant number should ideally be equal to one. For most pipeline systems it is impossible to satisfy exactly the Courant requirement with a reasonable (and common) Δt because they have a variety of different pipes with a range of wave speeds and lengths L [1]. There are at least three strategies to deal with this problem. The first strategy is apply the “method of the wave−speed adjustment” where one of the pipeline properties is altered (usually wave speed) to satisfy exactly the Courant condition; the second strategy is alter the reach length (modifying L and / or the number of reaches N); and the third strategy is leave everything unchanged and apply an interpolation procedure in pipes with $C_n < 1.0$.

2. GOVERNING EQUATIONS OF THE TRANSIENT FLOW

When analyzing a volume control it is possible to obtain a set of non−linear partial differential equations of hyperbolic type valid for describing the one−dimensional (1−D) transient flow in pipes with circular cross−section [2]:

$$\frac{\partial H}{\partial t} + \frac{a}{c}\frac{\partial Q}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial Q}{\partial t} + a \cdot c \cdot \frac{\partial H}{\partial x} + R\, Q|Q| = 0 \qquad (2)$$

Where: equations (1) and (2) correspond to the continuity and momentum (dynamics), respectively. Besides, ∂ = partial derivative, H = piezometric head, a = wave speed, c = g A / a, g = gravity constant, A = pipe cross−section, Q = fluid flow and R = f / 2DA, with f = friction factor (Darcy−Weisbach) and D = pipe diameter. The terms x and t denote space and time dimensions, respectively. Equations (1) and (2), in conjunction with the equations related with the boundary conditions of specific devices, describe the phenomenon of wave propagation for a water hammer event.

3. WAVE SPEED

For water (without presence of free air or gas) the more general equation to calculate the water hammer wave speed magnitude in one−dimensional flows is [3, 4]:

$$a = \frac{\sqrt{K / \rho}}{\sqrt{1 + \frac{K}{E}\frac{D}{e}\psi}} \qquad (3)$$

With K = volumetric compressibility modulus of the liquid; ρ = liquid density; E = pipe elasticity modulus (Young); e = pipe wall thickness and ψ = factor related with the pipe supporting condition. Equation (3) supposes that:

- Pipe has a thin internal wall, condition which is met when: D / e > 40 [4] or when D / e > 25 [5].

- Pipe remains full of water during the transient event; that is, no separation of the water column is generated, which means that at all times the pressure is greater than the vapour pressure.
- Water has small air content, so that the magnitude of the wave speed may be assumed constant.
- The pressure is uniform across any section of the pipe. It means that inertial forces associated with radial motion of the fluid are negligible [6].

Equation (3) includes Poisson's effect but neglects the motion and inertia of the pipe. This is acceptable for rigidly anchored pipe systems such as buried pipes or pipes with high density and stiffness, to name only a few. Examples include major transmission pipelines like water distribution systems, natural gas lines, and pressurized and surcharged sewerage force mains. However, the motion and inertia of pipes can become important when pipes are inadequately restrained (unsupported, free−hanging pipes) or when the density and stiffness of the pipe is small [7].

4. METHOD OF THE CHARACTERISTICS (MOC)

The Method of the Characteristics (MOC) is an eulerian numerical scheme [8] very used for solving the equations which governing the transient flow because it works with "a" constant and, unlike other methodologies based on finite difference or finite element, it can easily model wave fronts generated by very fast transient flows. MOC works converting the computational space (x) − time (t) grid (or rectangular mesh) in accordance with the Courant condition. It is useful for modelling the wave propagation phenomena in water distribution systems due to its facility for introducing the hydraulic behaviour of different devices and boundary conditions such as valves, pumps, reservoirs, etc. [9]. MOC has some main advantages, highlighting its ease of use, speed and explicit nature, which allows calculate the variables Q and H directly from previously known values [5, 10]. The main disadvantage of the MOC is that it must to fulfil with the Courant stability criterion that can limit the magnitude of the time step (Δt) common for the entire network. The MOC stability criterion states that [4]:

$$C_n = \frac{a\,\Delta t}{\Delta x} = \frac{a\,N\,\Delta t}{L} \le 1.0 \tag{4}$$

In order to get $C_n = 1.0$, some pipe initial properties can be modified (length and/or wave speed). Another way is to keep the initial conditions and apply numerical interpolations with risk of generating errors (numerical dissipation and dispersion) in the solution [11]. In general, MOC gives exact numerical results when $C_n = 1.0$; otherwise, it generates erroneous results in the way of attenuations (when $C_n < 1.0$) or numerical instability (when $C_n > 1.0$).

5. MOC: APPROXIMATE SOLUTION

MOC works projecting equations on "characteristic planes" whose traces on the position−time plane are called "characteristic lines", whereby a system of ordinary differential equations is achieved. If the convective terms are neglected an approximate solution is obtained [4]:

$$\frac{dQ}{dt} \pm c\,\frac{dH}{dt} + R\,Q|Q| = 0 \tag{5}$$

Eq. 5 is valid on the characteristic lines:

$$\frac{dx}{dt} = \pm\, a \tag{6}$$

Equations (5) and (6) exactly represent the system form by the basic equations, although limited to the subspace defined by the characteristic lines. When the sign + is taken, the positive characteristic equation C^{+} is obtained; with the sign – the negative characteristic equation C^{-} is obtained. The negative and positive characteristic lines are those through which the pressure wave propagates either upstream or downstream, respectively, of the form:

$$C^{+}:\ Q_P = Q_L + c\,H_L - (R\,\Delta t)\,Q_L\,|Q_L| - c\,H_P \tag{7}$$

$$C^{-}:\ Q_P = Q_R + c\,H_R - (R\,\Delta t)\,Q_R\,|Q_R| + c\,H_P \tag{8}$$

With Q_P = fluid flow at node P; H_P = piezometric head at node P and H_L, Q_L, H_R and Q_R are known state variables in nodes L and R, respectively (see figure 1). In border sections 1 and (N+1) an additional boundary condition is required, which must be solved in conjunction with the negative or positive characteristic equation depending on whether the first or last pipe sub−section, respectively.

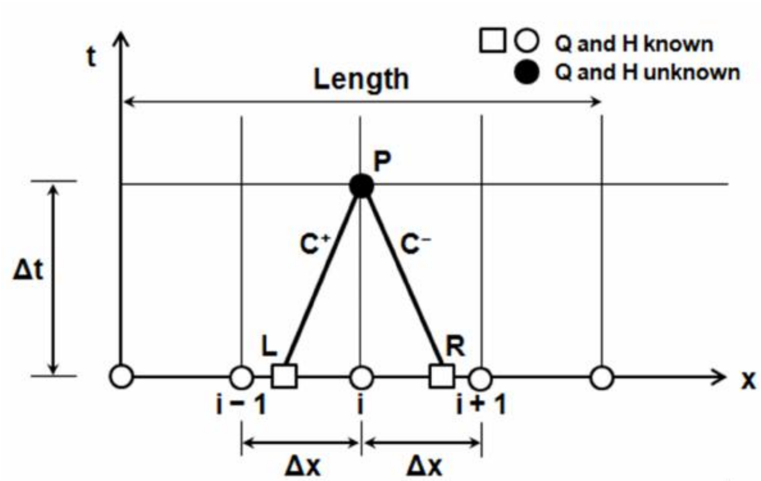

Figure 1. Space−time grid (Δx, Δt) with characteristic lines (C^{+} and C^{-}).

6. NUMERICAL INTERPOLATION

When MOC is applied with $C_n < 1.0$ some numerical interpolation must be applied in order to obtain Q and H for every pipe inner section. The most common numerical interpolation methods include linear interpolation at a fixed time level, including both space line interpolation and reach−out in space interpolation, as well as interpolation at a fixed location, such as time line interpolation or reach−back in time interpolation [1]. On the other hand, some authors [12] present a flexible discretization procedure computationally efficient which considers a variety of interpolation techniques, as well as of these approaches with the wave−speed adjustment technique, can be readily selected. When the interpolation is applied on x axis (figure 1), some analytical expressions can be obtained for state variables Q and H at interior nodes using numerical schemes with different interpolation orders [13]. For example, when Newton−Gregory method with interpolation order equal to 1 is used, the following equations are obtained when "i" varies between inner sections 2 and N [10, 13]:

$$H_L = H_i^t + (H_{i-1}^t - H_i^t) \cdot C_n \tag{9}$$

$$Q_L = Q_i^t + (Q_{i-1}^t - Q_i^t) \cdot C_n \tag{10}$$

$$H_R = H_i^t + (H_{i+1}^t - H_i^t) \cdot C_n \tag{11}$$

$$Q_R = Q_i^t + (Q_{i+1}^t - Q_i^t) \cdot C_n \tag{12}$$

With H_i^t, Q_i^t, H_{i-1}^t, Q_{i-1}^t, H_{i+1}^t and Q_{i+1}^t the state variables of internal nodes i, i − 1 and i + 1 at time t, respectively (figure 1). There is a tendency among practitioners to think of interpolation as a numerical device with only numerical side effects. In general, all common interpolation procedures result in numerical dissipation and dispersion, and they considerably distort the original governing equations and effectively change the wave speed [12]. In summary, interpolation fundamentally changes the physical problem and must be viewed as a nontrivial transformation of the governing equations.

7. SECTIONING FOR PIPING SYSTEMS: METHOD OF WAVE−SPEED ADJUSTMENT

In piping systems Δt must equal for all pipes. This involves a certain amount of care in its selection. It is quickly realized that equation (4) probably cannot be exactly fulfilled in most systems. Inasmuch as the wave speed is probably not known with great accuracy, it may be permissible to adjust it, slightly, so that integer N may be found. In equation form this can be expressed as [5]:

$$\Delta t = \frac{L}{a_j\,(1 \pm \phi_j) \cdot N_j} \tag{13}$$

In which ϕ_j is a permissible variation in the wave speed in pipe j, always less than some maximum limit of say 0.15 or 15% [5]. Despite the obvious this kind the adjustment takes with the physical problem, this procedure is widely recommended in the pipe literature [1].

8. PIPE LENGTH ADJUSTMENT

In general, a slight modification in wave speed is more preferable than any alteration in pipe length to satisfy the requirement of a common time step size [5]. Nevertheless, some authors [14] indicate that pipe lengths can be adjusted in the model so each pipe will be a length−wave speed combination such that the pressure wave will traverse the pipe in a time which is an exact multiple of the computational time increment. The pipe segment length tolerance should be the maximum difference between adjusted pipe lengths in the model and actual system, being a typical value 6 m [8].

9. EXAMPLE OF APPLICATION

Figure 2 shows the diagram of a very simple pipe network, which consists of a reservoir with H_0 = 100 m (upstream), a pipe (L = 4,800 m, flow rate Q_0 = 2.632 m^3/s, diameter D = 2 m, a_0 = 1,200 m/s and friction factor f = 0.022) and a valve (downstream) with a time of closure T_c = 35 (s). The steady state flow was solved using software EPANET [15]. In this case, the pipe network was discretized using N = 10 and Δt = 0.2 (s), values which when are replace in equation (4) we have: C_n = 0.5. In order to get C_n = 1.0 each option mentioned above will be applied (ceteris paribus) in the following paragraphs.

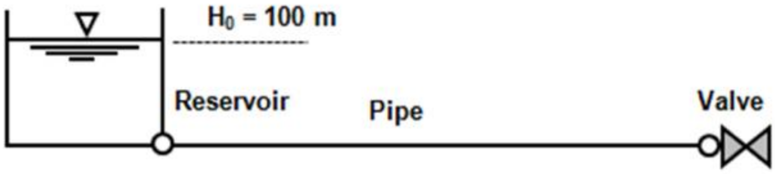

Figure 2. Pipe network example.

9.1 Wave−speed (a_0) adjustment

In order to get C_n = 1.0, the magnitude of a_0 must be increased by 100% up to a_1 = 2,400 (m/s). In addition to delivering a wrong solution by MOC, this option is an impossible situation because the theoretical maximum wave speed in pipe networks is approximate equal to 1,440 (m/s). Figure 3 shows the maximum and minimum pressure envelopes for this case (a_0 with continuous line, a_1 with dotted line), where the solution with a_1 generates significant errors compared with the exact solution.

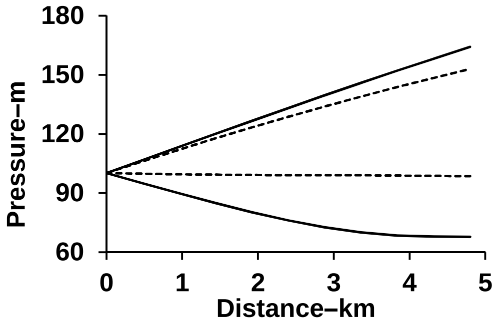

Figure 3. MOC: pressure envelopes (max. and min.) when a_0 = 1,200 m/s (continuous line) is incremented up to a_1 = 2,400 m/s (dotted line).

9.2 Number of reaches (N) adjustment

In this case N_0 = 10 must be doubled up to N_1 = 20 to get C_n = 1.0. With the doubling of N_0 an almost exact solution is obtained (figure 4). Nevertheless, this option becomes uneconomic from a computational point of view due to the increasing demand for memory and the slow rate of convergence to the solution: the program becomes two times slower.

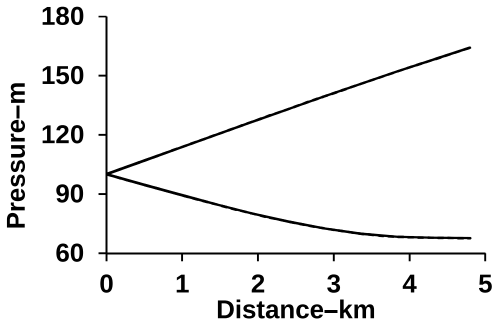

Figure 4. MOC: pressure envelopes (max. and min.) when N_0 = 10 (continuous line) is incremented up to N_1 = 20 (dotted line).

9.3 Pipe length (L) adjustment

The reduction of L_0 = 4,800 (m) to L_1 = 2,400 (m) permits to obtain C_n = 1.0, although the solution obtained corresponds to a very different problem to the posed originally (figure 5).

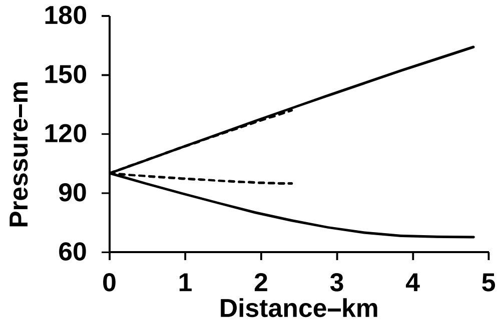

Figure 5. MOC: pressure envelopes (max. and min.) when L_0 = 4,800 m (continuous line) is reduced up to L_1 = 2,400 m (dotted line).

9.4 Application of interpolation on the axis−x

An alternative way of solution is to leave unchanged the initial data and uprightly applying an interpolation procedure on the axis−x. Figure 6 shows the results obtained where the appearance of attenuations is evident (and expected) when C_n < 1.0.

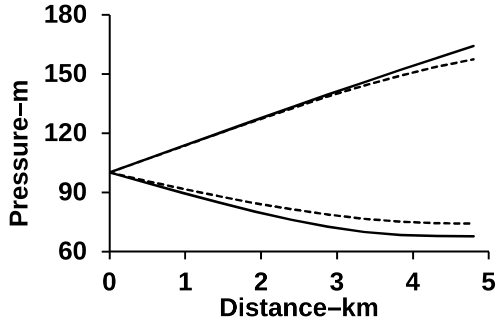

Figure 6. MOC: pressure envelopes (max. and min.) when numerical interpolation on axis−x is applied (continuous line: C_n = 1.0; dotted line: C_n = 0.5).

10. ALTERNATIVE TO THE MOC: HYBRID METHOD

A Hybrid Method (MH) is a multi–directional scheme that combines the best positive characteristics of two or more numerical schemes to produce a new different scheme capable of achieving a positive synergistic effect. Hybrid methods have been used to reduce (or eliminate) some problems associated with the stability of some numerical schemes such as the MOC, finite difference and finite element method. The general idea behind of HM is to use MOC for calculating the state variables at the pipe boundary nodes. For solving the internal nodes of each pipe, the Box's scheme will be used due to its best numerical stability and lower dependency of the required time step. The main objective is to reduce the attenuation and numerical dispersion when MOC is applied as a single solution algorithm.

Some authors [1] proposes a comprehensive and systematic approach to model different boundary conditions (with constant or variable consumption, reservoirs, etc.) or hydraulic devices such as valves, pumps, etc., which is based on a compatibility equation valid for linking the hydraulic behaviour of all piping, consumptions or tanks connected to each network node by:

$$H_P = C_c - B_c \cdot Q_{ext} \tag{14}$$

Where C_c and B_c are known constants and Q_{ext} is the nodal consumption or flow rate that may be constant or a function of time. The importance of equation (14) is it possible to separate or decouple the pipelines of complex networks in each node, restoring the flow continuity and the piezometric head (H_P) in the node (when no storage or singular losses). In the HM's context, MOC function is to calculate H_P for each node of the network using equation (14) as a border condition. The calculation can be done regardless of the number of pipes discharging to (or from) each network node. HM works with equations of dynamics and continuity applied to each pipe according to Box's (or Preissman) scheme, as follows:

$$d_1 Q_i + d_2 Q_{i+1} - d_3 H_i + d_3 H_{i+1} + d_4 = 0 \tag{15}$$

$$-c_1 Q_i + c_1 Q_{i+1} + c_2 H_i + c_3 H_{i+1} + c_4 = 0 \tag{16}$$

Where: d_1, d_2, d_3, d_4, c_1, c_2, c_3 and c_4 are constants that depend on the physical characteristics of the system, previously calculated values of the state variables and the spatial and temporal discretization. Equations (14), (15) and (16) can be used to construct a band diagonal system of dimensions 2 X (N + 1) which can be efficiently converted into a tri−diagonal system and then solved using the Thomas algorithm [13]. Figure 7 show the result obtained by HM when C_n = 0.5 where the solution is more conservative and almost coincident with the exact result obtained by MOC with C_n = 1.0.

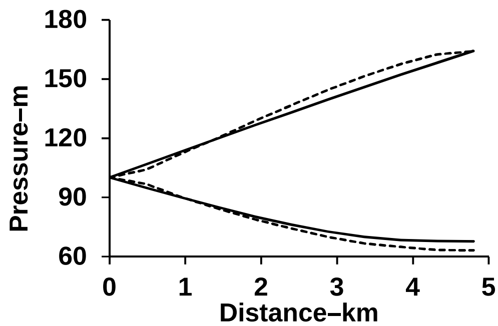

Figure 7. HM: pressure envelopes (max. and min.) when numerical interpolation on axis−x is applied (continuous line: MOC with C_n = 1.0; dotted line: HM with C_n = 0.5).

11. CONCLUSIONS

In the analyzed pipe network, very simple but numerically problematic, MOC gives an exact result only when the size of N is incremented by 100%. In this case it ceases to be effective the way of solution usually applied in the literature to "improve" the performance of MOC by modifying the initial physical data of the system (wave speed or length). At first glance the wave speed adjustment technique appears simpler because is non−dissipative and non−dispersive and in theory only consists in modify the value of the wave speed in a certain percentage to meet $C_n = 1.0$. Nevertheless, this procedure distorts the physical characteristics of the problem [12]. In other words, changing “a” involves altering, in physical terms, the value of one or more of the parameters that are part of its formulation, such as fluid density or the elastic modulus of the constituent element of the pipe. The modification of N may appear attractive, but it can mean a high cost in terms of computational memory when applied in large and complex pipe networks. In addition, equation (4) indicates that as N grows significantly decreases the value of Δt, causing the water hammer modelling becomes slower. The pipe length (L) adjustment can mean in some cases a minor modification. However, in the case analyzed L had to be reduced by 100% (up to 2,400 m) to make $C_n = 1.0$, a value that far exceeds the maximum value of modification recommended by some authors [8, 14]. The application of the numerical interpolation is the easier way to ignore the necessity of change anything in the system, but when $C_n < 1.0$ there are errors which appearing in shape of numerical attenuation. This leads to the need to raise other numerical schemes such as HM that are more stable and accurate than MOC when it is not possible to fulfil the Courant condition, and that do not require change any of the initial conditions in order to maintain their level of accuracy results. These required properties are important when it is necessary to solve the water hammer in large and complex pipe networks, with all kinds of pipes in terms of lengths, wave speeds, etc.

12. REFERENCES

[1] B.W. Karney, M.S. Ghidaoui, Flexible Discretization Algorithm for Fixed−Grid MOC in Pipelines, Journal of Hydraulic Engineering, (1997), 123(11), 1004-1011.

[2] M.H. Chaudhry, M.Y. Hussaini, Second-Order Accurate Explicit Finite−Difference Schemes for Waterhammer Analysis, Journal of Fluids Engineering, (1985), 107, 523-529.

[3] J. Twyman, Water hammer in a Water Distribution System, Paper accepted for publication in the Annals of the XXVII Latin American Congress of Hydraulic (IAHR AIIH), (2016), Lima, Perú, (in Spanish).

[4] G.Z. Watters, Analysis and Control of Unsteady Flow in Pipelines, Butterworth−Heinemann 2nd Edition, (1984), Boston. USA. ISBN 10: 0-250-40492-3.

[5] B.E. Wylie, V.L. Streeter, Fluid Transients, McGraw–Hill International Book Company. 1st Edition, (1978), USA, ISBN 10: 0-07-072187-4.

[6] R. Skalak, An Extension of the Theory of the Water Hammer, Columbia University, (1955), AFSWP No. 922, Tech. Report No. 15. pp. 37.

[7] M.S. Ghidaoui, M. Zhao, D.A. McInnis, D.H. Axworthy, A Review of Water Hammer Theory and Practice, Applied Mechanics Review, (2005), ASME, 58, 49-76.

[8] D.J. Wood, Water Hammer Analysis−Essential and Easy (And Efficient), Journal of Environmental Engineering, (2005), ASCE, 131(8), 1123-1131.

[9] R.O. Salgado, Revisión de los Métodos Numéricos para el Análisis del Escurrimiento Impermanente en Redes de Tuberías a Presión, XV Congreso Latinoamericano de Hidráulica, AIHR, AIIH, Cartagena, Colombia, 1992.

[10] M.H. Chaudhry, Applied Hydraulic Transients, Van Nostrand Reinhold, 1st Edition, New York, USA, 1979. ISBN 10: 0-442-21517-7.

[11] D.E. Goldberg, E.B. Wylie, Characteristics Method using Time−Line Interpolations, Journal of Hydraulic Engineering, (1983), 109(5), 670–683.

[12] M.S. Ghidaoui, B.W. Karney, Equivalent Differential Equations in Fixed−Grid Characteristics Method, Journal of Hydraulic Engineering, (1994), 120(10), 1159- 1175.

[13] J. Twyman, Decoupled Hybrid Methods for Unsteady Flow Analysis in Pipe Networks. Ed. La Cáfila, 1st Edition, (2004), Valparaíso, Chile. ISBN 10: 956-8142-17-7.

[14] D.J. Wood, S. Lingireddy, Pipe2012: Surge Modelling Tips and Procedures, pp. 30. 2012.

[15] L.A. Rossman, User’s Manual for EPANET, USEPA, Drinking Water Resources Division, Cincinnati, USA, EPA/600/R-00/057, 2000.