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IMPERFECT REPAIR MODELS FOR MULTIPLE REPAIRABLE SYSTEMS APPLIED TO RELIABILITY OF LOCOMOTIVES ELECTRONICAL COMPONENTS

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ABSTRACT

In this work, we present an application of classical reliability models in the study of the lifespan of electrical components present in a Brazilian logistics rail company. The main idea is that based on the failure history of different groups of electronic components, relevant information for the company's decision making can be ob- tained, such as the equipment lifetime behavior and and reliability prediction. At first, we present a brief introduction and quick development of the main concepts behind imperfect repair model and its ARA and ARI classes. For each group of com- ponents we estimate the models parameters, select the one with a better evaluation, analyse its the adequacy and present prediction results. The results obtained attest to the advantages of the imperfect repair models for the fit and interpretation of the studied dataset.

Keywords: Reliability analysis; maximum likelihood estimation; imperfect repair models; power law process; reliability predictor.

1 Introduction

One of the major problems faced by modern engineering is the occurrence of equipment failures in industries and factories. These failures occur naturally due to the wear of machine parts and components, transport vehicles and/or several other types of complex equipment that are essential for the core business of these companies. As a consequence, such failures lead to losses that include not only machine repair, but also delay in the production of products or provision of services and even loss of customers.

In this sense, the repair actions carried out soon after the occurrence of a failure are essential to avoid further damage to this type of company. Therefore, understanding the failure process and the impact of repairs performed on these equipment is essential for cost reduction and work optimization policies to be implemented.

This type of problem is well established in the area of Reliability, within the field of Statistics. In the reliability literature, systems that can return to normal operating conditions after a failure has occurred are called repairable systems. It is natural to think that complex systems used in industries and engineering companies are re- pairable, since it would not be feasible to discard them after a failure has occurred.

The theoretical scope used for repairable systems is naturally related to recurrent events, since successive failures can occur for the same system (details in [1]). In this sense, the statistical modeling for these recurrent events in repairable systems is done by counting processes [2]. In the literature of repairable systems, it is widely assumed that recurrent failures occur following a Non-Homogeneous Poisson Process (NHPP), and, in particular, one of its most important and known parametric forms is the Poisson Law Process (PLP), proposed by [3].

Moreover, the effect of the repair performed immediately after the failure is a crucial characteristic for the repairable system models and its definition is related to the system's failure process. The most widely known and discussed assumption in the literature is that after performing a repair, the system returns to the same condition as it was immediately before the failure, in a situation known as "as bad as old" (ABAO). This type of repair is called a minimal repair (MR) and more details can be found in works like [4] and [5]. On the other hand, there is also an assumption that the repaired system undergoes a renewal process, that is, after the repair, the system assumes a condition "as good as new" (AGAN). This type of repair is known as perfect repair and some works in the literature consider it as [6] and [7].

However, these two repair assumptions are not sufficient for real situations in the world of industry and engineering companies. It is expected that there are numerous situations where the repair will leave the system in an intermediate situation between AGAN and ABAO. This type of intermediate repair is called imperfect repair (IR) and it does not return the system to the same conditions as the PR, but leaves it in a better condition than a MR. The characteristic of wide applicability in real situations makes this type of repair a good assumption for modeling the failure times of repairable systems.

The motivation of this work is a real situation involving problems of failure of electrical and electronic components of locomotives of a Brazilian logistics company of rail transport. After failing, these components are sent to the company's maintenance laboratory, where they are restored and put back into operation. In this way, the objective of the study is to use the failure times of these equipment to understand the behavior of their lifetime and identify the efficiency of the repairs that are being performed. In this sense, the hypothesis that the repairs performed are IR seems to be plausible, since these equipment are not replaced and the repairs can improve the lifetime in some way. Furthermore, as will be discussed later, the models that involve IR are more general than the others and allow the quantification of the efficiency of the repairs performed.

Therefore, the goal of this work is to carry out a review of the literature about imperfect repair models and apply them to the modeling of real data involving failure times of a series of electrical and electronic components of locomotives. Frequentist methods will be used to estimate the parameters of the models presented. In addition, reliability predictors will be presented and applied in order to estimate the future behavior of the failure time of the systems under study, an extremely important result for the strategic organization of maintenance by industries and engineering companies.

The paper is organized as follows. In Section 2, we present the theory known in the literature about IR. In Section 2.1, we present some basic ideas about counting pro- cesses that underlie modeling theory for repairable systems; in Section 2.2 we present IR models based on the idea of virtual age presented by [8]; in Section 2.3 we present the ARA and ARI classes of the IR models presented by [9]; in Section 2.4 we present the modeling proposed by [10] for multiple repairable systems under IR condition, as well as the definition of reliability predictors taking into account this type of repair. In Section 3, we apply the models presented to fit five sets of data regarding the failure times of five different types of locomotive components. Finally, in Section 4, we high- light the main theoretical results presented in this work and their relevant application to a real situation and indicate some directions for future research.

2 Imperfect Repair Models

In this section we present a literature review of the theory of imperfect repair models. The idea is to present the base theory used for modeling systems under this type of repair so that they can be applied in the study of the central problem proposed in this work, in Section 3.
We start with the main ideas of counting processes to model the failure lifetimes, highlighting the non-homogeneous Poisson processes. Then, we discuss the first ideas of imperfect repair and virtual age defined by Kiijimas and the definition of imperfect repair classes ARA and ARI. Finally, we review the contributions of Toledo's work, which deals with estimation for multiple repairable systems under imperfect repa

2.1 Basic Assumptions

Let $N(t)$ be the number of observed failures of a single repairable system in the time interval $(0,t]$ and $0 < T1 < T_2$ a sequence of random variables that represent the failure times of this system and assume that the repair actions have negligible duration. Then the system failure process can be characterized by the two processes $\{N(t)\}_{t\geq 0}$ and $\{T(t)\}_{t\geq 0}$ and is completely determined by the *failure intensity function* given by

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\mathbb{P}[N(t,t+\Delta t] \geq 1 | \mathcal{H}_t]}{\Delta t} \qquad (1)$$

where $\mathcal{H}_t$ is the minimal filtration defined by the history of the process at time t and $N(t,t+\Delta t] = N(t+\Delta t) - N(t)$.

If process $\{N(t)\}_{t\geq 0}$ has independent increments, $N(0) = 0$ and the intensity function $\lambda(t)$is nonconstant, then the failure process is said to be a non-homogeneous *Poisson process* (NHPP). In this case, the *mean cumulative function* (MCF) can be equivalently called the *cumulative failure intensity function* and defined by $\Lambda(t) = \mathbb{E}[N(t)] = \int_0^t \lambda(u)du$.

Furthermore, this intensity function $\lambda(t)$ may or may not take a parametric form. One of the most known and used parametric forms for this type of process in the reliability literature is the Power-Law Process (PLP), where the failure intensity function and the cumulative intentity function are defined, respectively, by

$$\lambda(t|\beta,\eta) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \text{ and } \Delta(t|\beta,\eta) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta} \qquad (2)$$

with $t \geq 0$ and the parameters $\beta > 0$ and $\eta > 0$ are the shape and scale parameters,respectively. This parameterization proposed by [3] is defended by the authors of the area [11] because it is flexible, has an easy computational implementation and has a physical interpretation in the following sense: the parameter β indicates whether the system is improving ($\beta < 1$) or deteriorating ($\beta > 1$) over time, while the parameter η indicates the expected time so that only one failure occurs in the system (that is, $\mathbb{E}[N(\eta)] = \lambda(\eta) = 1$).

In the context of minimal repairs where the repair performed after the failure does not impact the system intensity function, if a PLP is assumed, the failure process is simply characterized by the expression $\lambda(t)$ in equation 1. In the context of imperfect repairs, this function will be discounted after each failure and its respective repair. However, until the first failure/repair the intensity function is the same, with the same behavior for any type of repair and, for this reason, it can be called the *initial failure intensity function* or simply the *initial intensity function*.

2.2 Virtual Age Models

Kijima [8] was one of the pioneers in the discussion of IR models. The idea of these authors was to propose a maintenance policy capable of balancing the prolonged times of deterioration of systems and the high cost of perfect repairs and/or replacement of these systems. The proposed model therefore introduced the concept of virtual age.
The main idea is that after repair, the system will assume an unreal "new age" that describes the current condition of the system compared to a new system. The virtual age is a positive function of the real age and the history of system failures, that is, $V_t = V(t|N(t); T_1, ..., T_{N(t)})$, where V_t represents the virtual age at time t.

More formally, let t be a failure time of a system, $V_{N(t)}$ its virtual age and $N(t)$ the number of failures/repairs at the time. If $V_{N(t)} = v$, the time $X_{N(t)+1}$ between the $N(t)$-th and the $(N(t)+1)$-th failures of the system works according to

$$\mathbb{P}[X_{N(t)+1} = x|V_{N(t)} = v] = \frac{F(x+v) - F(v)}{1 - F(v)} \tag{3}$$

where $F(x)$ is the lifetime distribution of a new system.

In this sense, X_n is the additional age after the $(n-1)-$th failure. However, the idea of the proposed model is that there is an efficiency in the repairs performed, which will consequently reduce the additional age X_n. In other words, a measure $a_n \in (0, 1)$ that represents the repair efficiency can be multiplicatively included in the model, reducing X_n and defining a virtual age to the system.

In work [12], Kijima et al. defined two IR models, depending on how the repairs performed affect the virtual age of the system. In the first model, the n-th repair affects only the n-th age increment X_n, and in this case the repair effect an reduces the increment X_n to $a_n X_n$, so that the virtual age after the n-th failure and repair is defined by $V_n = V_{n-1} + a_n X_n$. On the other hand, in the second model, the n-th repair affects the current virtual age V_n of the system and, in this case, the effect an reduces the virtual age V_n, which will now be defined by $V_n = a_n(V_{n-1}+X_n)$. Note that in both models, if $a_n = 1$ for all $n \geq 1$, the particular case of MR is obtained since $V_n = \sum_{n\geq 1} X_n$ for all n, whereas if $a_n = 0$ for all $n \geq 1$, the particular case of PR is obtained since $V_n = 0$ for all n.

2.3 The ARA and ARI Classes of IR Models

Doyen and Gaudoin proposed in their work [9] two new classes for IR models based on how the repair effect can reduce the model's initial intensity function. The first, ARA class (Arithmetic Reduction of Age) based on the reduction on the virtual age of system and the second, ARI class (Arithmetic Reduction of Intensity) based on the proportional reduction on the intensity function of the system. The authors use the idea of virtual age and repair effect from Kijima et al. and propose generalizations of the models presented above. Both classes proposed by Doyen and Gaudoin are defined by a memory m, which indicates that each repair action reduces the system wear that occurs after the last m failures. The memory m can be interpreted as the maximum number of previous failures that impact the effect of an actual repair. Note that the intensity function of the system is recalculated after each failure, considering the m most recent failure times.

Regarding the repair effect, in these models the authors consider that it is constant for all repairs performed. This effect is measured by a parameter θ, such that $0 \leq \theta \leq 1$ as in the models by Kijima et al..

2.3.1 The ARA$_m$ class model

The principle of the ARA$_m$ class, according to [9] is to consider that repair rejuvenates the system such that its intensity at time t is equal to the initial intensity at time Vt, where $Vt < t$. The failure intensity of an ARA model can be written as a function of its virtual age, that is, $\lambda_{ARA_m}(t) =_\lambda (V_t)$, where $\lambda(V_t)$is the initial intensity function.

Considering the repair effect parameter θ and a memory m, the failure intensity function for a model of the ARA$_m$ class is defined as

$$\lambda_{\text{ARA}_m}(t) = \lambda\left(t - (t-\theta) \sum_{p=0}^{min(m-1, N(t)-1)} \theta^p T_{N(t)-p}\right) \tag{4}$$

Two important particular (and extreme) cases of the ARA$_m$ class are the ARA1 and ARA$_\infty$ models. If $m = 1$, the failure intensity function for the ARA$_1$ model is given by

$$\lambda_{\text{ARA}_1}(t) = \lambda(t - (1-\theta)T_{N(t)}) \tag{5}$$

On the other hand, in the class ARA_∞ it is assumed that each repair reduces the virtual age of the system by an amount proportional to its age immediately before the repair. In this case, the failure intensity function is given by

$$\lambda_{\text{ARA}_\infty}(t) = \lambda\left(t - (t-\theta)\sum_{p=0}^{N(t)-1}\theta^p T_{N(t)-p}\right) \tag{6}$$

The particular class ARA_1 corresponds to the second virtual age model proposed by Kijima et al. in [8]. In this case, the MR and PR models are particular cases when $\theta = 1$ or $\theta = 0$, respectively.

2.3.2 The ARI_m class model

The basic idea of the ARI_m class model is to consider that each repair reduces the intensity of failure, depending on the failure history of the process. Thus, the failure intensity function of the ARI_m model, given the repair effect parameter θ and the history of the last m observed failures in the system, is defined as

$$\lambda_{\text{ARI}_m}(t) = \lambda(t) - (1-\theta)\sum_{p=0}^{min(m-1,N(t)-1)}\theta^p\lambda(T_{N(t)-p}) \tag{7}$$

where λ_t is the initial failure intensity function of the process.

Analogous to the ARA class, in the ARI_m model we can consider the existence of two extreme cases, ARI_1 and ARI_∞. If $m = 1$ the failure intensity function for the ARI_1 model is given by

$$\lambda_{\text{ARI}_1}(t) = \lambda(t) - (1-\theta)\lambda(T_{N(t)}) \tag{8}$$

In the ARI_∞ class, each repair reduces the intensity of the failure by a proportional value to the intensity of the current failure, in a cumulative sense of intensity reduction since the first repair. In this case, the failure intensity function is given by

$$\lambda_{\text{ARI}_\infty}(t) = \lambda(t) - (1-\theta)\sum_{p=0}^{N(t)-1}\theta^p\lambda(T_{N(t)-p}) \tag{9}$$

2.4 Estimation for Multiple Repairable Systems

Toledo et al. [10] used the models defined by Doyen and Gaudoin and proposed infer- ential methods for estimating their parameters considering both ARA and ARI classes from the failure histories of multiple repairable systems. In addition, in their work the authors also proposed a graphical goodness-of-fit analysis and a reliability prediction estimator from the best fit model. Below we present the main points of these ideas.

2.4.1 Parameters Estimation

Let us consider that k independent repairable systems are under observation for a predefined time t_k^* , with $k = 1, 2, ...$ Let $0 < t_{i,1} < t_{i,2} < \cdots < t_{i,n_i}$ be the observed failure times of the i-th system, where $t_{i,1} \leq t_k^*$ represents the time of the j-th failure in the i-th system, with $i = 1, ..., k$ and $j = 1, ..., n_i$. In this case, n_i is the number of observed failures of the i-th system and $N = \sum_{i=1}^k n_i$ is the total number of failures observed in k systems.

Considering that the initial intensity function $\lambda(t)$ is described by a parametric model and that the k systems are subjected to imperfect repairs with effect $\theta \in [0, 1]$ after each failure, we will denote by μ the vector of model's parameters, which includes the parameters of the initial intensity function and the repair efficiency parameter θ.

According to [10], the likelihood function for the parameter vector μ in the ARA_m and ARI_m models is given, respectively, by

$$\begin{aligned} L_{ARA_m}(\mu|\mathbf{t}) =& \prod_{i=1}^{k}\left\{\prod_{j=1}^{n_1}\left[\lambda_0(t_i, j-(1-\theta)s(t_{i,j}-1))e^{-\Lambda_0(t_{i,j}-(1-\theta)s(t_{i,j-1}))}\right.\right. \\ &\left.\left.\times e^{-\Lambda_0(t_{i,j}-(1-\theta)s(t_{i,j-1}))}\right]e^{-\Lambda_0\left(t_i^*-(1-\theta)s(t_{i,n_i})\right)+\Lambda_0\left(t_i-(1-\theta)s(t_{i,n_i})\right)}\right\} \end{aligned} \tag{10}$$

$$\begin{aligned} L_{ARI_m}(\mu|\mathbf{t}) =& \prod_{i=1}^{k}\left\{\prod_{j=1}^{n_1}\left[\lambda_0\big(t_i, j) + (1-\theta)\underline{s}(t_{i,j-1})\big)e^{-\Lambda_0(t_{i,j}+\lambda_0\left(t_{i,j-1}-(1-\theta)s(t_{i,j-1})\right)}\right]\right. \\ &\left.\times e^{-\Lambda_0(t_i^*)+\lambda_0\left(t_{i,n_i}+(t_i^*-t_{i,n_i}(1-\theta)\underline{s}(t_{i,n_i})\right.}\right\} \end{aligned} \tag{11}$$

where $\mathbf{t} = (\mathbf{t_{1,1}}, ..., \mathbf{t_{k,n_k}})$ denotes the vector of all the observed failure times and

$$\begin{aligned} s(t_{i,l}) &= \sum_{p=0}^{min(m-1,l-1)} \theta^p t_{i,l-p} \\ \underline{s}(t_{i,l}) &= \sum_{p=0}^{min(m-1,l-1)} \theta^p \lambda_0(t_{i,l-p}) \end{aligned} \tag{12}$$

Assuming that the initial failure process of the systems follows a PLP, the intensity and cumulative intensity functions given in (1) can be substituted in the (4) and (5) equations, obtaining a parametric likelihood function for the ARA_m and ARI_m models, respectively. In addition, the log-likelihood functions for the ARA_m and ARI_m classes can also be calculated and are respectively given by

$$\begin{aligned} l_{ARA_m}(\mu|\mathbf{t_{i,j}}) =& N(\log(\beta) - \beta\log(\eta)) + (\beta - 1)\sum_{i=1}^{k}\sum_{j=1}^{n_i}\log\left(t_{i,j} - (1-\theta)s(t_{i,j-1})\right) \\ &- \frac{1}{\eta^\beta}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left[\left(t_{i,j} - (1-\theta)s(t_{i,j-1})\right)^\beta - \left(t_{i,j-1} - (1-\theta)s(t_{i,j-1})\right)^\beta\right] \\ &- \frac{1}{\eta^\beta}\sum_{i=1}^{k}\left[\left(t_i^* - (1-\theta)s(t_{i,n_i})\right)^\beta - \left(t_{i,n_i} - (1-\theta)s(t_{i,n_i})\right)^\beta\right] \end{aligned} \tag{13}$$

$$\begin{aligned} l_{ARI_m}(\mu|\mathbf{t_{i,j}}) =& N(\log(\beta) - \beta\log(\eta)) + \sum_{i=1}^{k}\sum_{j=1}^{n_i}\log\left(t_{i,j}^{\beta-1} - (1-\theta)\underline{s}(t_{i,j-1})\right) \\ &- \frac{1}{\eta^\beta}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left[t_{i,j}^\beta - t_{i,j-1}^\beta - \beta(t_{i,j} - t_{i,j-1})(1-\theta)\underline{s}(t_{i,j-1})\right] \\ &- \frac{1}{\eta^\beta}\sum_{i=1}^{k}\left[t_i^* - t_{i,n_i}^\beta - \beta(t_i^* - t_{i,n_i})(1-\theta)\underline{s}(t_{i,n_i})\right] \end{aligned} \tag{14}$$

where $\underline{s}(t_{i,l}) = \sum_{p=0}^{min(m-1,l-1)} \theta^p t_{i,l-p}^{\beta-1}$

Given the complexity of the log-likelihood functions (6) and (7), the maximum likelihood estimates $\hat{\beta}$, $\hat{\eta}$ and $\hat{\theta}$ of the parameters β, η and θ must be obtained by maximizing these functions using numerical methods, as pointed out by [10]. Furthermore, the asymptotic properties of the maximum likelihood estimators based on the Normal distribution are used to construct the confidence intervals of the parameters.

2.4.2 Reliability Predictor

With the parameter estimates of the IR models, it is possible to estimate the reliability prediction of a system from the last observed failure time $T - n = t_n$ and observe the expected behavior of the lifetime of this system. Our interest is to estimate the time $t = T_{n+1} - t_n$ until the next failure considering the history $\mathcal{H}_{t_n}$ until the last observed failure t_n. The reliability prediction function at time t is expressed as

$$\begin{aligned} R(t) =& \mathbb{P}[T_{n+1} - t_n > t|\mathcal{H}_{t_n}] \\ =& \mathbb{P}[N(t_n + t) - N(t_n) = 0|\mathcal{H}_{t_n}] \\ =& \exp\left\{ - \int_{t_n}^{t_n+t} \lambda(u)du \right\} \end{aligned} \tag{15}$$

where $\lambda(t)$ is the intensity function of the model and $t_n \leq u \leq t_n + 1 < T_{n+1}$.

For the ARA_m class, substituting the intensity function $\lambda(t)$ in equation (8) by the intensity function λ_{ARA_m} given in (2), we obtain the predictor

$$\begin{aligned} R_{ARA_m}(t) =& \exp\left\{ - \int_{t_n}^{t_n+t} \lambda_{ARA_m}(u)du \right\} \\ =& \exp\left\{ - \frac{1}{\eta^\beta}\Big[(t_n + t - (1-\theta)s(t_n))^\beta \right. \\ &\left. + (t_n - (1-\theta)s(t_n))^\beta\Big] \right\} \end{aligned} \tag{16}$$

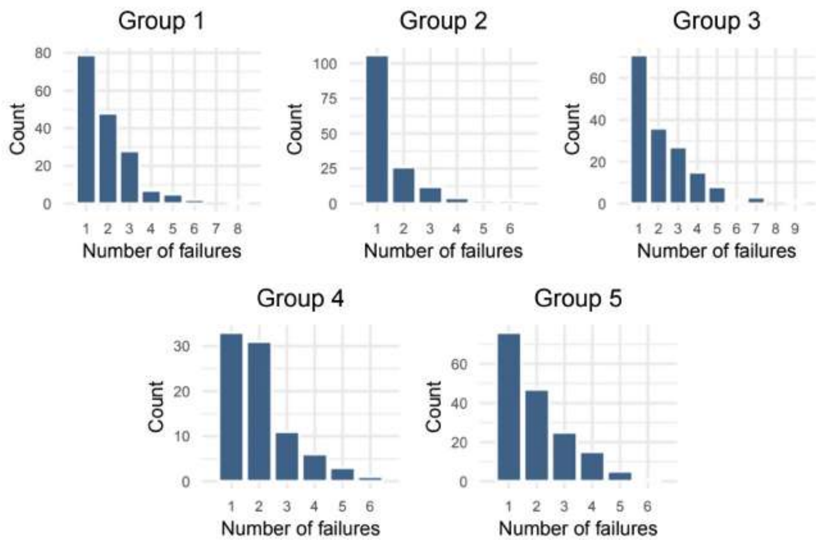

Figure 1: History length distribution of the pieces presented in the dataset according to groups of components.

In the same way, for the ARI_m class we introduce the intensity function λ_{ARI_m} given in (3) into the equation (8) and we obtain the predictor

$$\begin{aligned} R_{ARI_m}(t) &= \exp\left\{ -\int_{t_n}^{t_n+t} \lambda_{ARI_m}(u)du \right\} \\ &= \exp\left\{ -\frac{1}{\eta^\beta}\Big[(t_n+t)^\beta - (t_n)^\beta \right. \quad (17) \\ &\qquad \left. - t(1-\theta)\beta\underline{s}(t_n)\Big] \right\} \end{aligned}$$

Furthermore, from the reliability prediction function we can obtain the *mean time to failure* (MTTF) at the time $Tn = t_n$, that is, the expected time to the next failure occurring after a time t_n for a given system. The MTTF at the time t_n is given by

$$\text{MTTF}_{t_n} = \mathbb{E}[T_{n+1} - t_n | \mathcal{H}_{t_n}] = \int_0^\infty R(t)dt \quad (18)$$

where $R(t)$ is given by (9) and (10) for the ARA_m and ARI_m classes of IR model.

3 Real Data Applications

The dataset used in this work was obtained from a Brazilian logistics rail company that has a whole mechanism of different types of equipment. Each unit registered in the software has a repair history, followed by its technical problems and the evaluation of the technicians during repair.

In this case, it is relevant to study the quality of the repairs based on system failure history to help the technicians in the decision making process of verifying if a piece should be officially discarded or if it is worth to be refixed given its reliability predictor. The dataset is composed by five classes of different unit types. For each one of these groups it was assigned a unique model for their future lifespan predictions.

Figure 1 shows the distribution of the number of observed fails for each equipment group. It is clear that the mean number of repairs from each unit is relatively low for all groups. That is, a unit will contain at max 9 fails followed by its repairs.

Each data point will contribute for the information in the likelihood function from two separate cases: one of them involves handling the information about the actual fail times occurred during the unit lifetime and the other one concerning the time between the data sampling and the piece's last fail. The latter indicates the need of the application of reliability models truncated in time t^*, which represents the time during which the unit was still working when the dataset was obtained.

For each groups of equipment we tested three dis-

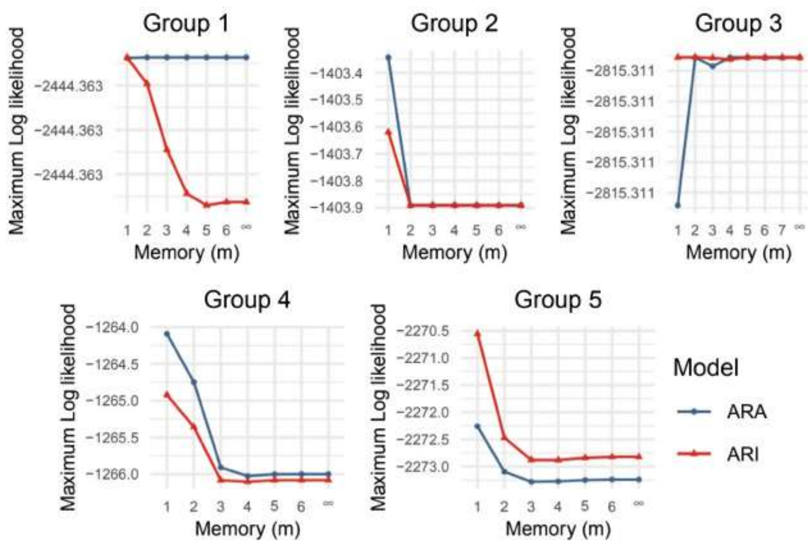

Figure 2: Log likelihood maximum estimate for group 5.

tinct model approaches, based on the type of repair considered. One of these models is the minimal repair model (MR), that considers that once a unit is repaired, its intensity function doesn't suffer an actual reduction, keeping the piece's instant fail risk the same as before it broke. The other two models correspond to classes ARA_m and $\text{ARI}_m$of IR, and, in these cases, all possible failure memories m were tested for each group.

First of all, it is necessary to select the best memory m to fit the data from the ARA_m and ARI_m models and the criterion used for this selection is simply the one with the highest log-likelihood. The results for all groups and both classes are shown in Figure 2. In this case, for almost all the groups, it happened when $m = 1$ for both ARA and ARI models. This is an interesting result from the dataset, and is justified by the large proportion of components that have only one failure during the observed period, as shown in Figure 1. For the groups 1, 2, 4 and 5 it was clearly observed a decrease in the quality of the fit with the increasing value for m. On the other hand, there is an increasing pattern of the maximum log-likelihood maximum only on group 3, precisely the group in which more failures were observed for a single system. In addition, the expected stability of maximum log-likelihood when the memory m value tends to infinity [13] were observed for all the five groups.

After choosing the best fits of the ARA and ARI classes, these fits were compared with the fit by the MR model. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) were used to decide which of the models resulted in a better fit of the data. The estimation results by the three models for each equipment group is given by Table 1. In addition, the best model for each group is marked by $*$ in Table. The results presented in Table 1 allow some conclusions about the lifetime of the components and the repairs performed.

Initially, note that the estimate $\hat{\beta}$ is greater than 1 in all groups, indicating that the systems are degrading over time. As stated in Section 2.1, the estimate $\hat{\eta}$ indicates the expected time for exactly one failure in the system, so if we consider, for example, a component of Group 1, it is expected that one failure occurs in approximately 3758 days.

The AIC and BIC criteria attest that the groups 1, 2 and 3 were best explained by the MR model, suggesting the unit repair does not result in a decrease of the instant risk function. In groups 4 and 5, on the other hand, the repair effect parameter θ shows suggests a slightly decrease in the intensity function, representing an actual effect on the repair efficiency. Note that in these last two groups the 95% CI do not indicate an approximation to MR or PR, reaffirming the existence of a repair effect impacting the intensity function.

Table 1: Parameters estimation results for each of the six groups.

Group	Model	$\hat{\beta}$ (95% $CI_{\hat{\beta}}$)	$\hat{\eta}$ (95% $CI_{\hat{\eta}}$)	$\hat{\theta}$ (95% $CI_{\hat{\theta}}$)	AIC	BIC
	ARA_1	2,30 (2.04, 2.55)	3758.06 (3493.70, 4022.42)	1 (0.80, 1)	4894,73	4906,96
1	ARI_1	2,30 (2.03, 2.56)	3758.06 (3500.49, 4015.64)	1 (0.74, 1)	4894,73	4906,96
	MR^*	2,30 (2.05, 2.54)	3758.06 (3522.37, 3993.76)	—	4892,73	4900,88
	ARA_1	2,76 (2.37, 3.15)	3465.04 (3228.40, 3701.68)	0.89 (0.70, 1)	2812,69	2823,59
2	ARI_1	2,72 (2.35, 3.10)	3482.34 (3241.23, 3723.45)	0.86 (0.52, 1)	2813,24	2824,14
	MR^*	2,68 (2.32, 3.04)	3523.11 (3300.64, 3745.59)	—	2811,78	2819,05
	ARA_1	2,08 (1.84, 2.32)	2947.46 (2721.86, 3173.06)	1 (0.75, 1)	5636,62	5649,21
3	ARI_1	2,08 (1.86, 2.30)	2947.49 (2718.83, 3176.15)	1 (0.74, 1)	5636,62	5649,21
	MR^*	2,08 (1.87, 2.29)	2947.49 (2752.48, 3142.50)	—	5634,62	5643,02
	ARA_1^*	2,46 (2.05, 2.88)	2444.53 (2222.00, 2667.06)	0.57 (0.41, 0.74)	2534,18	2544,48
4	ARI_1	2,33 (1.99, 2.68)	2460.06 (2219.19, 2700.92)	0.48 (0.24, 0.72)	2535,85	2546,15
	MR	2,09 (1.76, 2.42)	2681.28 (2431.47, 2931.09)	—	2543,79	2550,66
	ARA_1	1,61 (1.41, 1.80)	1657.66 (1493.87, 1821.45)	0.64 (0.33, 0.95)	4550,52	4562,66
5	ARI_1^*	1,61 (1.43, 1.78)	1598.60 (1427.61, 1769.59)	0.69 (0.48, 0.89)	4547,11	4559,25
	MR	1,53 (1.36, 1.70)	1728.6 0(1567.17, 1890.03)	—	4551,49	4559,59

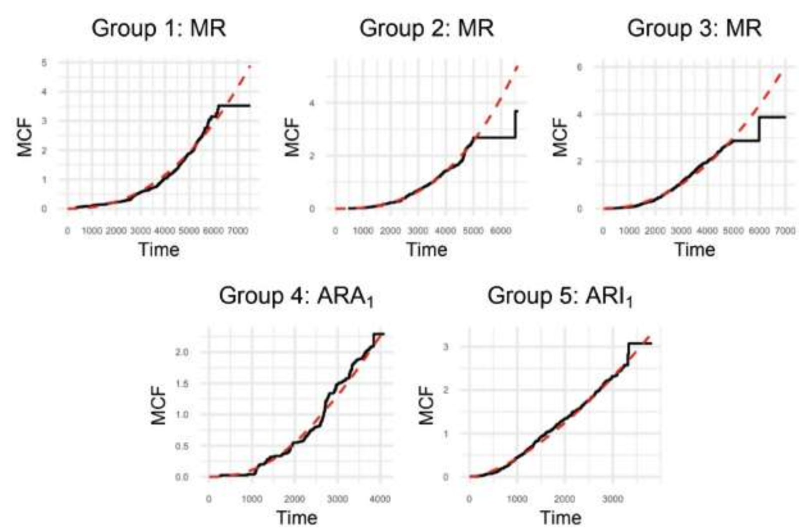

Figure 3: History length distribution of the pieces presented in the dataset according to groups of components.

For the suitability tests of the five groups it was used mainly the graphical con- struction proposed by [10], using the empirical MCF values from the data compared to a mean of the estimation from the parametric models. As pointed out by [7], the non-parametric estimation of the MCF is obtained based on the Nelson-Aalen proce- dure and the goodness-of-fit plot comparison is that the better the fit the closer the empirical MCF will be to that estimated by the model. As shown in Figure 3 it was observed an overall good fit of the models for all groups studied.

Based on the fitted parameters, each individual unit can be described by the selected models. Figure 4 shows the predicted reliability functions for the units in each group with the most number of fails and repairs. Taking the unit observed in group 1, for example, we can see that the reliability probability for 7500 days after the last failure is very close to zero, given the entire history of the failure process.

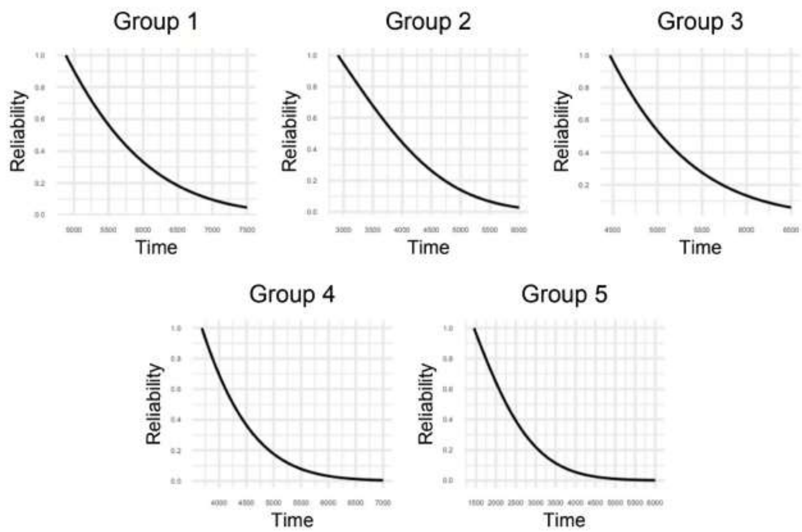

Figure 4: History length distribution of the pieces presented in the dataset according to groups of components.

Table 2: Mean estimated times until next failure in components with the most number of fails in each group, in days.

Unit	1	2	3	4	5
Mean	949.71	1138.28	774.14	821.57	1053.92

Finally, using the equation (11), the mean estimated lifetime from the last repair of these units were calculated and are given in Table 2.

4 Conclusion

In this paper we presented the main concepts of reliability analysis for repairable systems subjected to imperfect repairs after each failure. A literature review about IR models was performed and inferential methods for estimating their respective parameters and predicting reliability were established.

An interesting new dataset were presented and analyzed in the context of the two classes ARA and ARI of the IR models. Some obtained results are not commonly seen in the literature, in which the increasing memory of the imperfect repair model in fact leads to a reduction of the quality of the model. As mentioned before, the probable cause for this effect is the fact that there are many units with a single observed fail, supporting the concept of using an inferior memory value m for the ARA_m and ARI_m classes.

In the five groups of failure times observed, all of them could be reasonably described by the parametric models as the plots on the empirical MCF values suggests. As a result, it was possible to make predictions for the units failure times using directly the models selected for this task.

The results of this paper attest to the applicability of IR models, since the objective of modeling and making predictions on a real dataset of failure times of multiple repairable systems was achieved. Based on these results, the models used could contribute for a more complex investigation of the data, for example, to study the minimization of the maintenance cost when a fail occurs and to search ways to indicate the disposal of pieces that show low predictions for their remaining lifespans. In this sense, as a proposal for future work, our research group intends to revisit the dataset presented in this paper to improve the analyzes under these new possibilities.

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